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Stability of the Market Economy in the Presence of Diverse Economic Agents

Anjan Mukherji

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JICA Research Institute
10-5 Ichigaya Honmura-cho
Shinjuku-ku
Tokyo 162-8433 JAPAN
TEL: +81-3-3269-3374
FAX: +81-3-3269-2054

Stability of the Market Economy in the Presence of Diverse Economic Agents

Anjan Mukherji*

Abstract

The stability of market economy is defined and stability conditions deduced which do not appear to restrict preferences in any significant manner. This assumes importance when considering economies where diversity among agents is known to exist. It is shown that if a condition on the rank of the Jacobian matrix of the excess demand functions at equilibria is satisfied then equilibria will be locally asymptotically stable. When this condition is not met, it is shown how redistributing resources may lead to stable competitive equilibrium. It is also shown how instead of imposing credible penalties, which may cause significant incentive problems, redistributing resources may serve to provide the correct incentives to agents who otherwise might have contributed to market failure.

Keywords: Stability of equilibrium, diverse preferences, redistribution of resources, rank condition, incentives for competitive transactions

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* Centre for Economic Studies and Planning Jawaharlal Nehru University (amukherji@gmail.com)

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1 Introduction

The functioning of the market economy deserves special attention today particularly because all economic policies which are currently in place all around the world stem from the theory of the market economy. There has never been any time in recent history that the basic paradigm was so uniform. Consequently it is perhaps not so surprising that when things have started to malfunction, the spread of that phenomenon has been also more or less uniform across countries; it is basically a symptom of our times that when some malaise attacks a country or its economy, its spread to other parts of the world is certain. The effects may vary from economy or country to country but the impact is surely felt. Witness for example the spread of phenomenon which has been called a financial crisis: its spread across the world, covering all countries has been attributed to the fact that these days all countries follow the same paradigm economically.

A legitimate point of enquiry would then be whether it is this paradigm which is at fault. Traditionally when markets fail to deliver the desirable outcomes, markets are said to fail. We thus seek to enquire whether the failure in the market observed recently may be attributed to some thing that economists have missed: some thing which was required to be in place for their policy to bear fruit but which in fact was not. Given the huge canvas that we have to tackle we shall make our task a bit easier by confining attention to only one aspect of the functioning of the market economy viz., its stability and whether the diversity of the economic agents has anything to do with its failures. We shall also try to investigate whether there is any thing that one can do about this aspect.

Since this is not going to be an essay in the History of Economic Thought, we shall not provide a comprehensive development of the various concepts that we shall deal with. On the contrary, we shall provide the current interpretation or definition of the various notions that we need to introduce. We shall proceed as follows: we first define what economists mean by a market economy; whether anything special is meant by the use of this term. We shall then consider what is so attractive about such an economy and why economists of various persuasions have been enamored with the functioning of the market economy. And we shall see what economists mean by the term stability of the market economy; we will then be able to investigate what economic theory has to say about the stability of the market economy and how diversity of economic agents or decision makers may create problems for such a notion. Finally we shall provide a theoretical analysis of what needs to be done to adjust the results of economic theory to function when there is diversity.

This of course implies that we believe that this diversity does affect the results of economic theory. Economic decision makers or agents are generally identified by their preferences or tastes and their purchasing power: we shall assume that on both counts there are considerable differences. For the purposes of our discussion this is what we shall mean by the term diverse economic agents: economic agents with diverse preferences or tastes. While this diversity is at the basis of all economic transactions, the theory developed has always rested on some aspects of commonality of preferences: witness the widespread use of the representative agent model. These are descriptions where one set of preferences is supposed to adequately capture the tastes of set of decision makers under study. It is indeed difficult to see why such a set-up will be useful in analyzing any economic problem. Clearly this emphasis, if there is one, indicates that theory based on such models is likely to be useless while treating situations with diverse agents.

2 The Market Economy

The phrase market economy actually means a competitive market economy: the first term being dropped and taken for granted. What this means is an economy with a group of agents who are competitive that is who are unable to control the price, at which trading occurs, to their own advantage. Thus the market price is determined through forces of demand and supply without any agent being able to exert control. In fact any situation where the agents act as if they are competitive would result in such an economy. Why should one actually find such an economy at all attractive? The reason for the attraction lies mainly in two results which go by the name of The Fundamental Theorems of Welfare Economics. In addition to appreciate the true nature of the market economy, we have a result known as the Equivalence Theorem. These three aspects together constitute the main basis why competitive market economies have been analyzed and followed so closely. To discuss these results we need to introduce the notion of a competitive equilibrium. At the heart of a competitive economy lies the notion of an equilibrium, or more properly a competitive equilibrium which describes a configuration where all the plans made by various agents match. Thus a set of markets with prices where every demand from a buyer is matched by supply from a seller would constitute such a competitive equilibrium. The results mentioned above relate to such an equilibrium. The important thing about a competitive equilibrium configuration is that it is possible to conceive of such a configuration even though there may be many diverse economic agents, making plans according to their own tastes to buy and sell in each market. Furthermore the First Fundamental Theorem of Welfare Economics testifies to the efficiency properties of such a configuration. Efficiency refers to a very special property viz., among all feasible configurations there is no other alternative where some agent is better off and no one is worse off. That competitive equilibrium achieves such a desirable state of affairs is indeed noteworthy. The Second Fundamental Theorem considers the class of all efficient configurations and deduces that for any such configuration, it is possible to find prices such that if initial resources were distributed appropriately then the prices would form a competitive equilibrium at which the efficient configuration would be realized; this was thus a converse to the previous result.

Amongst economists of various political beliefs, after all economists too are diverse, the appeal of the above results differed. Amongst those who believed that the free market was the panacea for all that was wrong, it was the First Fundamental theorem which was the main result. For if the market was not efficient then it would be possible to reorganize matters to make some one better off and leave no one worse off and hence to ensure efficiency, free markets were required. There were those of somewhat different persuasion who believed that efficiency by itself was not desirable and that only some efficient state was desirable: these were the social planners or socialists, in short; they believed in the Second Fundamental Theorem. But they too believed in the ability of the market to attain the competitive equilibrium. The third result we have referred to as the Equivalence Theorem provides another interesting aspect of the competitive equilibrium.

The competitive equilibrium, it may be recalled provides a configuration of markets and prices at which demand and supplies match and all plans made by the various decision makers are realized simultaneously. And a new distribution of resources results at the end of trading. If the objective of the theory of competitive markets was to provide a method of arriving at an alternative distribution of resources which is acceptable to all, then we may consider other possible methods or processes to arrive at the same end. Consider the following process: each decision maker comes to the negotiation place with a listing of all resources each currently controls. There is a Negotiator who totals up the total stock of all resources and suggests an allocation; agents may object if any one of the two hold: a)Any agent feels that he/she is worse off compared to the initial situation or if b)Any group of agents can withdraw with their own resources and from this stock can provide to each member of the group an allocation which leaves no one worse off and some one better off when compared to the Negotiators suggestion.

This of course is radically different manner of proceeding: in this agents do try to control what gets accepted; moreover, they are allowed to form groups and even veto suggested allocations. If the redistribution passes this test and is not objected to, the redistribution is said to belong to the Core. Clearly allocations in the Core are the only acceptable allocations. The Equivalence Theorem asserts that Core and Competitive Equilibrium allocations coincide provided there are many agents. Thus the redistribution achieved through a competitive equilibrium market transaction cannot be objected to in the above manner. Clearly, a very desirable property indeed.

The point of departure for this essay comes at this juncture: granted that the competitive equilibrium is a useful notion with desirable properties, what are the conditions which would guarantee that a competitive equilibrium will be established. There are two aspects to this query:

- Under what conditions will the market be able to arrive at the equilibrium configuration? And

- What is the guarantee that at the market equilibrium, agents carry out their competitive market transactions?

If we are unable to find answers to these questions or if we find answers which are difficult to ensure then no matter how attractive the notion of a competitive equilibrium maybe, we shall have to give up the notion of such an equilibrium being even a benchmark. For then since there are no guarantees that competitive equilibrium will ever be achieved even theoretically, why should one concern oneself with its very nice properties?

We shall attempt to take up these important questions for closer scrutiny in the pages below and as we hope to establish, these two aspects may require some preconditions and since these pre-conditions are never checked for, hoping to arrive at a competitive equilibrium is unlikely to materialize and consequently the failures of the type we have noted at the beginning may be easier to understand. Her Royal Highness, the Queen of England, at a recent ceremony in the London School of Economics, asked the galaxy of economists present why in spite of the huge grants made by Her Majestys Treasury to support research, economists were unable to forecast the crisis or avert it. Apparently some of her loyal subjects are debating what the answer to that very legitimate query should have been[1]. As we shall see, the current generation of economists, regardless of their political persuasion took for granted certain aspects. In fact these are the things

[1]As far as I understand there have been two sets of responses but while one set of economists blame the emphasis on formal mathematical treatment in economics for the debacle they were at a loss to indicate why they themselves were unable to pinpoint what was wrong or what was coming. Or maybe because they have always shouted wolf, no one took them seriously; but they will be surely unable to analyze why the turn around takes place. It is not mathematics per se or formal treatments which are to blame; as we shall see what is to blame is the inability of both sets of economists to analyze fully the set of conditions under which the policies based on the market economy work.

which are first responsible for the current mess and unless we appreciate these matters fully, we will be unable to take any adequate corrective steps. However, we have strayed a bit from our objective in this current paragraph. To return to our basic queries, posed above we shall begin by stating what is meant by the phrase **stability of the market economy**.

3 Stability of the Market Economy

The competitive market was supposed to solve for the equilibrium prices by itself. In fact the famous 'Invisible Hand' was supposed to be able to achieve this and this idea is generally mistakenly attributed to Adam Smith. It is only later writers who have attributed this power to the Invisible Hand[2]. Not only were writers wrong about the source of this belief, they appear to have been mistaken in their belief that the Invisible Hand was successful in attaining an equilibrium. We shall be concerned with the latter aspect in this paper. As we have remarked, when subjected to scrutiny, this belief in the Invisible Hands power, did not hold up and conditions under which this was possible, the so-called 'stability conditions' needed to be invoked. The working of the Invisible Hand was through the forces of demand and supply, it may be recalled; consequently the need for stability conditions implied that just demand and supply did not possess the power to achieve this target.

The formalization of the role of demand and supply appeared through the specifica-

[2] See for instance a commentator as profound as Frank Hahn (1982 a). Since the term Invisible Hand was thought to be coined by Adam Smith in the celebrated book Wealth of Nations, the role of the Invisible Hand in equilibrating markets is some times attributed to Adam Smith; but Smith mentions Invisible Hand once in History of Astronomy, for the first time, completed around 1758 and then in his book The Theory of Moral Sentiments (1759) and then in the Wealth of Nations (1776). In addition, the reference in the last was made not while discussing markets in Books I and II but only in Book IV where Smith was advocating support of domestic industry over foreign! So while we shall use the term Invisible Hand in our paper, it should be noted that the failures or successes of this instrument should not be attributed to Adam Smith but rather to those who thought that he said so and followed this bit of fiction blindly.

tion that price moves in the direction of excess demand that is demand minus supply: thus if demand exceeds supply in any market, the price will be bid up while if demand is less than supply then the price in that market will be bid down. Formally[3], while the intuition says that price adjustment should be in the direction of excess demand, a simplification is often used whereby the price adjustment is taken to be proportional to the level of excess demand. At the equilibrium excess demands are zero and price adjustment too stops. The question that we seek to examine is whether beginning from any arbitrary price configuration initially, the price adjustment equations generate a solution which approaches the equilibrium: If the answer is yes than we say that the market economy is stable whereas if this is so only under some additional restrictions then we say that these conditions, which enable us to conclude that the solution approaches the equilibrium, are stability conditions.

That stability of market economy could not be taken for granted was first noted by Gale (1964) and Scarf (1960). Their exercise consisted of setting up a class-room type example of a market economy: a one market economy involving two persons and a two market economy with three persons in the case of latter; specification of tastes and resources available led to the construction of demand functions; supplies were assumed fixed since what was being studied was just the exchange process. And it was found that the price adjustment in the direction of excess demand need not necessarily lead to the equilibrium. Thus stability, it was implied, was a special property.

Around the same time as these examples were being investigated, work was also progressing on another front: namely finding out conditions on excess demand functions which led to stability that is identifying stability conditions. Basically these stability

[3] We are using a simple form of these equations where the price change is proportional to the level of excess demand and the constant of proportionality is unity. This simplifies exposition considerably and choosing the factor to be unity is not of significance. What is a significant restriction is to choose the price adjustment to be proportional to excess demand; the intuition is basically that the rate of price change in any market should have merely the same sign as excess demand. See however Mukherji (2008) on the justification for choosing the rate of price adjustment to be a constant proportion of the excess demand.

conditions were in the nature of restriction on preferences or tastes of decision makers or agents: See for example, Negishi (1960) and Hahn (1982) for surveys of this area. It was noted too that had the market demand originated from the maximization of a single welfare function or if tastes were similar to the extent that net buyers and net sellers behaved similarly, stability of equilibrium could be ensured. It would therefore appear that if preferences were diverse, which is the setting for this exercise, these conditions may be difficult to ensure. And consequently, to ensure stability, we could no longer rely on preferences being restricted in some manner. Thus alternative avenues needed to be explored.

Indeed as we shall see, without restricting preferences in any manner, one could still obtain stability of equilibrium by redistributing resources. This is a meaningful exercise in the present context, since preferences are not restricted in any manner and if required, redistributing resources appear to be a meaningful exercise. The implications of the necessity for such a course of action may not be evident immediately and we shall return to this later. For the moment, we investigate this phenomenon in some detail.

3.1 A Stability Condition: its Violation and Restoration

Consider an economy with a single market where good x is exchanged against good y at the price p per unit i.e., each unit of y is treated to be equivalent to p units of good x. Assume too that supplies of the goods are fixed. Demands are obtained from each agent and aggregated to obtain market demand and consequently excess demand (demand supply). Plotting in a graph, excess demand (Z) against p with the former on the vertical axis and the latter on the horizontal axis, we may get a diagram as the one shown below.

Notice that $p^\star$, where $Z(p)$ intersects the horizontal axis, is such that excess demand is zero: demand = supply and all plans made by every agent match; not only that for p¿p*, we can see that excess demand is negative or that demand is less than supply and hence under our usual intuition says that the price will be revised back towards equilibrium; hence stability of equilibrium is equivalent to the excess demand curve

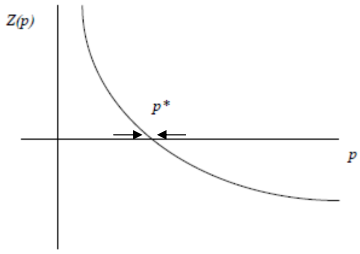

Figure 1: A Stable Equilibrium

being downward sloping at equilibrium. If this condition is satisfied the Invisible Hand works well!

In the case of Gale (1963), the excess demand curve is shown to be upward sloping at equilibrium:

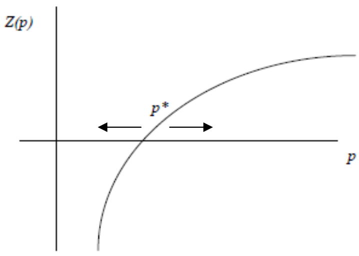

Figure 2: The Gale Example

Notice that now $p > p^\star$ implies that excess demand is positive and hence price is revised upwards and away from equilibrium. The Invisible Hand is woefully inadequate to perform its job. Thus Gale (1963) says "Arrow and Hurwicz have shown that for the case of two goods, one always has global stability...... Nevertheless, some queer things can happen even in this case."[4]

However, in this set up, we show in the appendix that if we change the resource distribution, the equilibrium remains unaltered but the excess demand returns to the shape

[4] A formal demonstration is contained in the appendix.

depicted in Figure 1, viz., a stable equilibrium results. Let us try to see why this may be so. One may note that, the redistribution leaves the equilibrium unchanged at p = 1, since the purchasing power has remained the same and hence so do the demands; but because the endowments have changed, the trades at equilibrium are different. Gales example led to instability because of the adverse net sellers income effect; by redistributing resources, the previous net seller became the net buyer at equilibrium and consequently the adverse income effect was turned around. Consequently, the Invisible Hand works well exactly as Adam Smith or rather Frank Hahn had said, it would and the "queer things" noted by Gale disappear.

It may be pointed out that the above finding at first glance, is somewhat contrary to results in the literature. Stability of equilibrium as we have mentioned above was usually associated with the preferences of individuals. In the above example, A and B have preferences which apart from being different, have a property that each person treats the goods to be complementary to one another: viz., with each unit of good x, 2 units of good y must be consumed, for example. Substitution is not possible. More importantly, income effects of price change are the only effects. And since substitution effects are known to be stabilizing and they were absent in the present context, it is to be expected that in Gale (1963), the equilibrium was unstable.

However when we change the initial distribution of resources, preferences do not change: there are no substitution effects; only income effects remain as before; yet the unique interior equilibrium is now globally stable[5].Thus we need to re-examine alternative stability conditions. This discussion seems to indicate that whether the Invisible Hand works or not, i.e., whether the equilibrium is stable or not, may depend upon the initial distribution of resources. If this were to be found correct then one of the preconditions for the working of the competitive market is that initial resources be properly distributed. We shall examine this matter next. It may be recalled that this was the first aspect that we had mentioned above.

[5]Mukherji (2000), (2002) and (2007) contain discussions of similar situations in the context of the example due to Scarf (1960).

3.2 A Stability Condition

Consider the following scenario then: two agents (I and II) meet to exchange two goods and arrive at a better distribution than what they had at the beginning. Let us also assume for the moment that the agents behave competitively and once again as in the discussion of the last section, that the supplies are fixed: whatever was available with the agents initially i.e., the endowment is fixed. We shall examine whether the equilibrium will be stable: the answer here will have to be in the negative given the example due to Gale (1963); and we shall investigate whether we can identify restrictions which will identify a stability condition. The situation then be described thus:

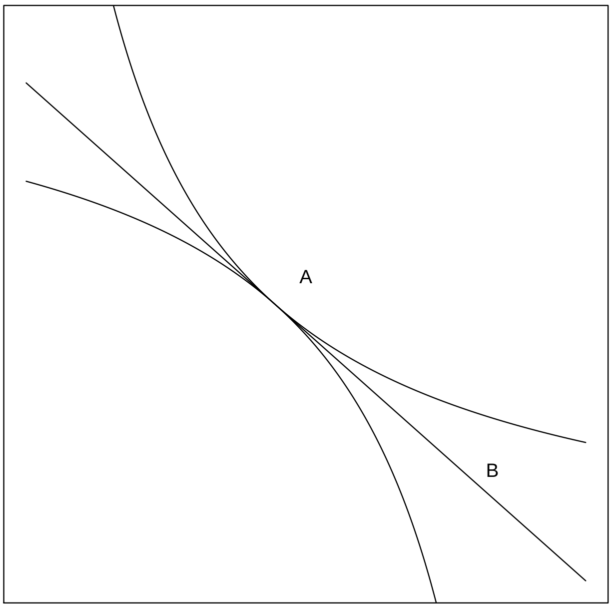

Figure 3: Competitive Equilibrium

We represent the exchange situation in terms of a Box diagram drawn above; the lower left hand corner is the point of reference for individual I while the upper right hand corner is the point of reference for individual II; the dimension of the box represents the available amounts of goods x,y available for distribution[6]. Point B say represents the

[6]For I, the amount available is measured with reference to lower left hand corner and the remaining

initial distribution. The competitive assumption means that there will be a price p represented by the slope of the straight line through the point B at which both I and II would wish to be at A and so we have a matching of the plans.

The question is whether the equilibrium is stable. As we have seen this may not be so. However again, as we have shown in the context of the Gale example, there is some redistribution of initial endowments, so that the new endowment lies somewhere along the straight line through B such that the equilibrium remains to be p and both agents continue to wish to be at A and finally the equilibrium p with the new endowment configuration is stable.. The point is whether this may be generalized to any number of goods and agents. The answer is in the affirmative provided the regularity condition (i.e., slope of excess demand curve with A as endowment has a non-zero slope at the equilibrium) is suitably defined and the demonstration of this fact is contained in the appendix.

We provide here the main intuition behind the result. Economic theory predicts that had the initial distribution been at A, the resulting equilibrium would be the trivial zero-trade equilibrium at p. But the excess demand function when initial distribution is at A will have very nice properties. A slight rotation around A to make the line steeper, signifying a higher value for p would mean that while I would wish to attain a bundle which is up and to the left of A, individual II would want a bundle to the right and downwards from A[7]. Notice that both I and II would therefore wish to sell good x. Thus at a price higher than equilibrium, there would be an excess supply (i.e., a negative excess demand) for the good and hence the price will be revised downwards. Consequently regardless of preferences, the excess demand curve, with A as endowment, would be negatively sloped at the equilibrium: a case of stable equilibrium[8].

Now suppose that when the initial distribution is at B, the equilibrium is unstable

amount for II, with reference to the upper right hand corner. Any point in the Box thus represents a distribution of the total amounts available.

[7]Here the terms up and down, right and left are to be interpreted with reference to the lower left hand corner.

[8]This result was first noted by Arrow and Hurwicz (1960).

with say the excess demand curve being positively sloped[9]. Shifting the distribution to A means that excess demand curves would be negatively sloped; hence somewhere between A and B, there is a distribution at which excess demand curve must have a zero slope at equilibrium. In other words, with the initial distribution somewehere in between A and B, the excess demand curve with the axes as specified above must be horizontal. Of course one is assuming that there is enough continuity and smoothness.

Now just as zero-trade equilibria are considered trivial; excess demand curves with zero slope at equilibrium are considered special, because the equilibrium may be dislodged by slight perturbation and hence models are defined to be regular, when excess demand curves are non-zero sloped at equilibrium. Notice that this means that for regular economies, where excess demand curve has a non-zero slope at every equilibrium, we cannot have an unstable equilibrium; this follows exactly as argued above. When excess demand curves at any equilibrium has a non-zero slope, instability now would necessarily mean a positively sloped excess demand curve at equilibrium; by redistributing resources, without changing the equilibrium, but making the equilibrium into a zero trade equilibrium, the excess demand curve has a negative slope; hence there would be some intermediate distribution of resources for which the excess demand curve will have a zero slope at equilibrium . Even if we do not impose regularity, it means that there is always the possibility of fixing the stability property by properly distributing resources and this need not involve removing all possibilities of trade at equilibrium.

The appendix provides the formal arguments in support of the above. In conclusion therefore, we need to worry about the distribution of resources : this is an aspect which has been uncovered mainly by our by-passing the traditional restrictions on preference structure while discussing stability properties.

[9] Notice that the excess demand curve is drawn with the price on the horizontal axis. Thus the slope of the excess demand curve is merely the slope of the tangent to the curve at p or the derivative of the function Z(p).

4 Incentives for Equilibrium Transactions

We turn next to the second query mentioned initially. At a competitive equilibrium configuration what incentives do agents have to carry out their competitive transactions. More specifically, if every one else is behaving competitively, should an agent behave competitively? If the answer is in the negative, we cannot continue to base economic policies on a competitive paradigm. Usually such a question is seldom asked and it is taken for granted that agents should behave competitively.

We consider this aspect, in the Appendix, through an example of exchange involving three persons and three goods. There are three individuals A, B, and C with utility functions and endowments involving three goods x, y, z as under

$$A : min(x, y); w_A = (1, 0, 0); B : y.z, w_B = (0, 1, 0); C : min(z, x), w_C = (0, 0, 1)$$

Thus although there are three goods each individual is interested in consuming only two goods and further, each has a stock of only one commodity. Further preferences differ. While A consumes only goods x, y in the fixed 1:1 proportion and C consumes goods z, x in a fixed 1:1 proportion, B consumes both goods y, z with rectangular hyperbola type indifference curves allowing smooth substitution among these goods provided by the utility function mentioned above. Notice too that each has one unit of the good that one is consuming.

Our first task is to compute the competitive equilibrium in this set-up. The steps are provided in the Appendix. The unique interior equilibrium is given by $p^\star = 1, q^\star = 1$. We look at the trades which take place at equilibrium, next. In accordance with their demands and supply the following transactions will clear markets.

A gets 1/2 units of good z from C in exchange for 1/2 units of good x; which is then traded to B for 1/2 units of good y.

We will like to investigate whether it is in the interest of these three to carry out the above transactions. Suppose now at the competitive equilibrium, B does not surrender 1/2 units of good y and say, surrenders only a somewhat smaller amount; if everyone else behaves competitively, then B ends up with the bundle 1/2 units of x and slightly more than 1/2 unit of y; thus we are trying to investigate whether B would try to go back on the commitment to trade at competitive equilibrium prices; notice that by deviating from competitive trades, and if B is not coerced in any manner, B would be better off than at the equilibrium and hence B will have an incentive to undersupply. And of course if there is no compulsion of any sort, then B will definitely undersupply and the competitive equilibrium will fail to materialize. What about the others? Notice that preferences or tastes are such that A and C cannot gain by undersupplying. Consider A for example: does A benefit from supplying less than 1/2 unit of x if every one else is behaving competitively? Since A consumes goods x, z in proportion 1:1 and receives 1/2 unit of z from C, having more than 1/2 unit of x does not increase his sense of wellbeing: A has thus no incentive to undersupply. Similarly C has no incentive to undersupply as well. But B will find it advantageous to appear as if B is agreeable to competitive behavior, exhort A, C to accept such a behavior and then renege on the agreement and sell short. This too should be considered to be a failure of the competitive market.

4.1 A Penalty Scheme to induce Competitive Behavior

Consider then the probability of B being undetected; let this be given by d and let this probability depend on the extent of undersupply5; if detected undersupplying, a penalty is imposed on B which depends once again on the extent of undersupply and will consist of a penalty in terms of units of good y. Does this deter B? We show in the appendix how such a penalty scheme will deter B from undersupplying and induce competitive behavior.

Basically, B will expect to be detected and fined with some probability ; so if the extent of the penalty and the probability of being detected is high enough, B would expect to be worse off from undersupplying and hence will find it advantageous to comply and act competitively. It may be shown that competitive behavior is seemingly ensured, only when the probability of being detected is high enough in comparison with the penalty rate.

What seems easy to state actually opens up an entire Pandora's box: regulation of competitive markets actually creates further problems, as a moment's reflection will clarify. Since regulators may lack proper incentives to carry out the task assigned to them, insisting on regulators may have unleashed a whole set of new and additional problems. Competitive behavior seems almost impossible to ensure in any meaningful way, even in the context of such a simple example.

4.2 An Alternative Method

Consider however the following option of redistributing endowments so that A has (0,1,0) and B has (1,0,0) while C continues to hold (0,0,1): notice that as in the case of the Gale example considered earlier, at the equilibrium, purchasing power has not changed; this implies that demands too have not changed; however, transactions have altered substantially.

The new equilibrium remains (1,1); we consider in the appendix, a detailed analysis of the new equilibrium and its stability properties to conclude that the new equilibrium is stable. But transactions at equilibrium have changed due to the changed endowments: B sells 1/2 x to A to get 1/2 y and sells 1/2 x to C to get 1/2 z; notice that now B too has no incentive to go back on the competitive transactions. This is so because what B has to sell, good x, has no direct value to B, given the nature of Bs utility function, and hence holding back units of this good has no attraction for B. There seems to be a moral in this tale. Thus a redistribution of endowments has succeeded where other measures such as regulation would have led to many other associated problems: creating more problems for the establishment of competitive transactions.

5 Conclusion

In an economy characterized by diversities of all kind, how is one assured that the market economy will function? In fact what reasons do we have to advocate that competitive markets will provide the ideal situation? Even setting aside the questions regarding externalities or asymmetric information when markets are known to fail, we point out two fundamental points of concern. The first relating to the stability of competitive equilibrium: if stability doesn't obtain the market may not gravitate towards a competitive equilibrium and hence we do have a kind of market failure. The second source of market failure arises when at a competitive equilibrium, agents lack the incentives to carry out the equilibrium transactions. Here the word "failure" is being used to signify that what obtains is not competitive equilibrium and hence may lack the properties we usually attribute to a competitive equilibrium state.

We suggest that in such a framework we cannot impose restrictions on preferences: the Weak Axiom of Revealed Preference in the Aggregate or WARP, for example, is obtained when there is either a single homogenous utility function whose maximization yields market demand or when tastes have properties characterized by the so-called Gorman (1961) form, for example. In such a framework and in the light of our experience with the examples of Instability due to Gale (1963) and Scarf (1960), we observed that one way out of this problem would be to examine the role of the distribution of endowments. This departure from the traditional way of approach has several advantages: if an useful condition could be formulated then its applicability to an economy with diversity would not be open to question. Moreover, in terms of policy prescriptions, it seems more plausible to state a property about the distribution of endowments rather than on the utility functions.

We deduce a stability condition and show how redistribution of endowments may lead to an equilibrium when this condition is violated. It should be pointed out that the last result is like the Second Fundamental Theorem of Welfare Economics since we show that there will be some distribution of endowments which will make a given price configuration stable. The details are provided in the section below.

Appendix

A1 The Gale Example

Consider the following example due to Gale (1963). There are two persons **A,B** with utility functions defined over commodities (x, y) as follows: $U_A(x, y) = \min(x, 2y)$ and $U_B(x, y) = \min(2x, y)$; their endowments are specified by $w_A = (1, 0), w_B = (0, 1)$; routine computations lead to the excess demand function of the first good (x), $Z(p)$, where p is the relative price of good x:

$$Z(p) = \frac{p - 1}{(p + 2)(2p + 1)}$$

Thus the unique **interior** equilibrium is given by $p = 1$[10]; now notice that if the adjustment on prices is given by

$$\dot{p} = h(p) \tag{1}$$

where $h(p)$ has the same sign as $Z(p)$ and is continuously differentiable so that the solution to (1) say $p_t(p^o)$ is well defined for any initial point $p^o > 0$.

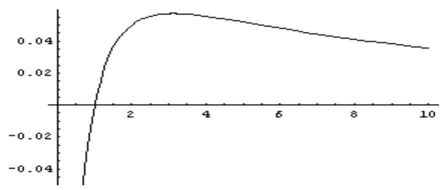

Figure 4: Excess Demand - The Gale Example

As Gale (1963)[11] says, " Arrow and Hurwicz have shown that for the case of two goods, one always has global stability...... Nevertheless, some queer things can happen

[10] There is an equilibrium at infinity.

[11] There are two sets of examples in this contribution; we consider here the two-good example. A treatment of the three good example is contained in Mukherji (1973); see also Bala (1997), in this connection.

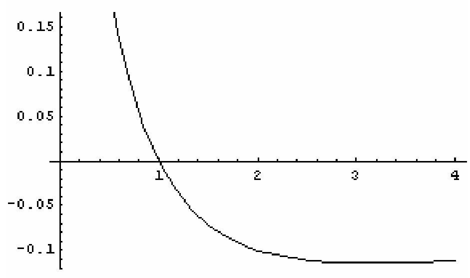

Figure 5: Gale Example with a switch in endowments

even in this case." To see the queer things referred to, consider the function $V(p_t) = (p_t - 1)^2$ and notice that along the solution to the equation (1), we have $\dot{V}(t) > 0$ for all t, if $p^o \neq 1$: so that the price moves further away from equilibrium and there is no tendency to approach the unique interior equilibrium.

Notice that the excess demand curve is upward rising at the interior equilibrium and hence we have the above conclusion.

However, in this set up, let us tinker with the distribution of resources. Suppose for example, we interchange the endowments i.e., **A** has $(0, 1)$ while **B** has $(1, 0)$. One may note that at equilibrium $p^\star = 1$, the purchasing power has remained the same and hence so do the demands but because endowments have changed the trades at equilibrium are different. Recomputing excess demand functions, we note that the unique interior equilibrium is now globally stable. This follows since the excess demand function is now:

$$Z(p) = \frac{2(1-p)}{(2p+1)(p+2)}$$

Notice now that the instability of the interior equilibrium noted earlier disappears. One may therefore say that we had instability of the interior equilibrium because the pattern of purchasing power, in relation to endowments had not been *right*. With the new pattern of endowments, excess demand curve becomes downward sloping. This should be the first indicator that for stability, an appropriate distribution of endowments may be essential. Notice too that this is necessary because individuals are not identical in either tastes or endowments and this is why such investigations assume importance.

It may be instructive to consider the Gale example in some further detail. We first considered the endowment distribution in Gale: $(1, 0), (0, 1)$ for A, B respectively; we then switched it to $(0, 1), (1, 0)$ for A, B respectively. Consider a weighted average of these two distributions $(\lambda, 1 - \lambda), (1 - \lambda, \lambda)$ to A, B respectively, where $0 \leq \lambda \leq 1$; thus for $\lambda = 1$, we have the Gale endowment pattern and for $\lambda = 0$ we have the switched pattern that we used to deduce Figure 2; notice that at $p = 1$ the purchasing power of the individuals remains the same at these distributions; consequently the demand does not change and hence $p = 1$ is an equilibrium for each such distribution; however the excess demand function changes. Routine calculations yield:

$$f(p, \lambda) \equiv Z_x = \frac{2(\lambda(p-1)+1)}{2p+1} + \frac{p + \lambda(1-p)}{p+2} - 1.$$

Consequently

$$Z_x = \frac{(p-1)(3\lambda - 2)}{(2p+1)(p+2)}$$

and hence

$$Z_{xp}|_{p=1} = (3\lambda - 2);$$

hence $p = 1$ for all values of $\lambda < 2/3$ is **stable**; when $\lambda = 2/3$, the derivative vanishes (in fact, $Z_x(p) = 0 \forall p$ if $\lambda = 2/3$) and consequently the rank condition (Assumption 7) is violated. Our choice of $\lambda = 0$ worked to stabilize the equilibrium but clearly as is evident, there are many other possible redistributions which will achieve the same end. The following diagram may clarify how changes in the values of λ alters the excess demand function.

Notice that the excess demands $f(p, 0)$ and $f(p, 1)$ were drawn earlier; $f(p, 2/3)$ is a horizontal through the point $(0, 0)$: if $\lambda < 2/3$ the excess demand is downward sloping at $p = 1$ while for $\lambda > 2/3$ the excess demand is upward sloping at $p = 1$.

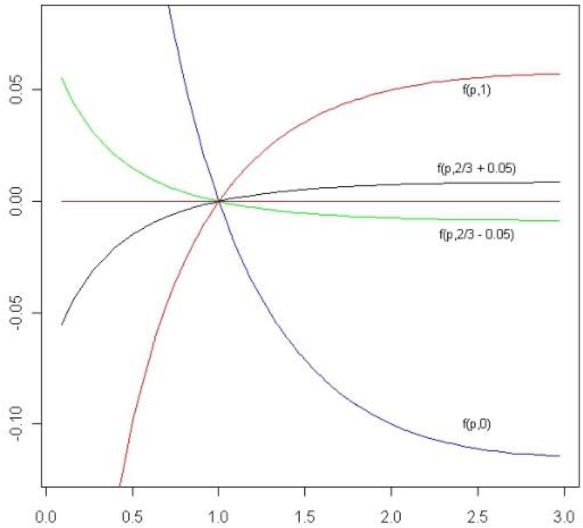

Figure 6: Excess Demands for alternative values of λ

A3 The Model

We shall assume as in Negishi (1962) that we are analyzing the standard exchange model involving m individuals and n goods and that the total amounts of these goods is given by the components of $\bar{W} \in \Re^n_{++}$; each individual has a real-valued utility function $U^i : \Re^n_+ \to \Re$; further each U^i is assumed to be *strictly increasing, strictly quasi= concave and continuously differentiable.* Sometimes, we shall specify a distribution of $\bar{W}$ among the individuals usually denoted by $w^i \in \Re^n_{++}$ such that $\sum_i w^i \leq \bar{W}$; let us denote the set of all such feasible allocations by the set $\mathcal{W}$; if an allocation $\{w^i\}$ has been chosen from $\mathcal{W}$, we can then proceed with defining demands $x^i(P)$ as the unique maximizer of U^i in the budget set provided by[12] $\{x : P^T.x \leq P^T.w^i\}$, where $P \in \Re^n_{++}, P = (p_1, \cdots, p_n)$ is the price vector; **in case** we have a numeraire, we shall consider good n to be the numeraire and write the price vector as $P = (p, 1)$; the vector of **relative prices** will then be written as $p \in \Re^{n-1}_{++}$.

[12]We shall use the superscript T to denote matrix transposition.

Market demands are defined by $X(P) = \sum_i x^i(P)$; excess demand is then defined by $Z(P) = X(P) - \bar{W}$. Strictly speaking we should write $Z(P, \{w^i\})$ however, we usually omit the distribution of the resources and write $Z(P)$.

Excess demand functions are expected to satisfy:

1. $Z(P)$ is a continuous function and bounded below for all $P > 0$;

2. Homogeneity of degree zero in the prices i.e., $Z(\theta P) = Z(P) \forall \theta > 0, P > 0$.

3. Walras Law i.e., $P^T.Z(P) = 0 \forall P > 0$;

   to these we add the following assumptions:

4. $Z(P)$ is continuously differentiable function of prices for all $P > 0$.

5. For any sequence, $P^s = (p_1^s, p_2^s, ..., p_n^s) \in \Re^n_{++}$, $p_i^s = 1, \forall s$ for some index i, say $i = i_o$ and $||P^s|| \to +\infty$ as $s \to +\infty \Rightarrow Z_{i_o}(P^s) \to +\infty$[13](Boundary condition).

   The above conditions are standard and all of them excluding the last, in fact appeared in Negishi (1962); the importance of the role of assumptions such as the last, (the Boundary condition), was realized somewhat later[14].

   Finally, the equilibrium for the economy, with individual resources $\{w^i\} \in \mathcal{W}$, is defined by $P^\star$ such that $Z(P^\star) = 0$. Under the assumptions mentioned above, we know that an equilibrium exists and the set $\mathcal{E} = \{P \in \Re^n_{++} : Z(P) = 0, \text{ for some } \{w^i\} \in \mathcal{W}\}$ is non-empty.

A4 The Tatonnement Process

Consider an allocation $\{w^i\} \in \mathcal{W}$; unless otherwise stated this allocation will be held fixed in this section. The price adjustment process which we shall study is the following:

$$\dot{p}_j = \gamma_j Z_j(P) \text{ for all } j, \gamma_j > 0 \tag{2}$$

[13] $||x||$ stands for $\sqrt{(x_1^2 + x_2^2 + ... + x_n^2)}$, when $x = (x_1, x_2, ..., x_n)$.

[14] One of the earliest in this connection was Arrow and Hahn (1971), Assumption 1, p. 293.

This is what Negishi called the 'non-normalized' system where the adjustment occurs on **all** prices.

A related process studied involves the choice of one good as the numeraire or the unit of account so that all prices are measured relative to good n; then the price vector is $P = (p, 1)$ and we may write $Z(P) = Z(p, 1) \equiv Z(p)$. The adjustment is then considered only on the relative prices:

$$\dot{p}_j = \gamma_j Z_j(p) \text{ for all } j \neq n, \gamma_j > 0 \tag{3}$$

This process is called the 'normalized' system. We shall consider mostly the **latter**.

Given our assumptions of the last section, for any initial price $P^o = (p^o, 1)$, there is a solution to (3) denoted by $\phi_t(p^o) = p(t)$, say. The equilibrium for the process is $P^\star = (p^\star, 1)$ such that $Z(P^\star) = Z(p^\star, 1) \equiv Z(p^\star) = 0$ and hence coincides with equilibrium for the economy, i.e., $P^\star = \Psi(\{w^i\}) \in \mathcal{E}$, where $\Psi : \mathcal{W} \to \mathcal{E}$ and thus associates with each distribution $\{w^i\} \in \mathcal{W}$ an equilibrium price configuration $P^\star \in \mathcal{E}$. Given that the choice of the numeraire remains fixed, we shall refer to $p^\star$ as the equilibrium for the economy when $P^\star = (p^\star, 1) \in \mathcal{E}$; we shall in such cases, we may refer to $p^\star \in \bar{\mathcal{E}}$, where $\bar{\mathcal{E}} = \{p : (p, 1) \in \mathcal{E}\}$.

Consequently, we need to investigate whether $\phi_t(p^o) \to p^\star \in \bar{\mathcal{E}}$ as $t \to +\infty$. The stability of competitive equilibrium examines this question.

We shall say that the *equilibrium* $p^\star$ *is globally stable under (3), if the solution* $\phi_t(p^o) \to p^\star$ *as* $t \to \infty$ *for any arbitrary* p^o*; if convergence is ensured only if* $p^o \in N(p^\star)$*, where* $N(p^\star)$ *is some neighborhood of* $p^\star$*, then we shall say that* $p^\star$ *is locally stable under (3).* We mention the following two results in connection with the process (3)[15]:

1 *Given the assumptions stated above, the solution to (3)* $\phi_t(p^o)$ *from any* p^o *with* $p_i^o > \varepsilon_i$ *remains within the positive orthant for all* t *and remains bounded away from the axes i.e., satisfies* $p_i(t) > \varepsilon_i$ *for all* $t > 0$.

[15]For Proofs, see Mukherji (2007)

In addition, we have:

2 *The solution to (3), $\phi_t(p^o)$, from any p^o with $p_i^o > \varepsilon_i$ remains within a bounded subset of $\Re^n_{++}$.*

In the above circumstances, we are assured that the solution to (3) has limit points within the positive orthant, provided that the initial price was strictly positive.

Why study such processes? We have analyzed this question in some detail in Mukherji (2008 a) and (2008 b). We showed that if the endowments are redistributed appropriately, then a process like (2) is the modified gradient process for attaining an optimum for the economy; moreover, this process always converges. The gradient process is always shown to converge; it may be identified with (2) only if the endowments are appropriately adjusted. *Thus price adjusting proportionally to the level of excess demand has some defense but only under the assumption that the distribution of endowments is proper. And when it is defensible, it works; that is, the solution converges.* The investigation into Gale and Scarf examples was the first indicator that the distribution of endowments has an important role to play. The gradient process and its properties is the second hint that we should be considering the role of the distribution of endowments. In Mukherji (2008), we had presented a regularity condition on the distribution of endowments which implied global stability of equilibrium. We shall present in the next section an extension of those results.

A5 Sufficient Condition for Stability of Equilibrium

It should be noted then that the excess demand functions not only depend on the price p but also on the distribution of endowments $\{w^i\}$ and we shall assume that

6. $Z_j(p, \{w^i\}), Z_{jk}(p, \{w^i\})$ for each j, k and any $p > 0$ are continuous in $\{w^i\} \in \mathcal{W}$.

Consider the matrix, the Jacobian of the excess demand functions defined as below:

$$J(p, \{w^i\}) = \begin{pmatrix} Z_{11} & \cdots & Z_{1n} \\ \cdots & \cdots & \cdots \\ Z_{n1} & \cdots & Z_{nn} \end{pmatrix}$$

where all the partial derivatives are evaluated at $(p, \{w^i\})$. By using the properties introduced above, we have the following:

3 *For any configuration* $(p, \{w^i\})$, $p > 0, \{w^i\} \in \mathcal{W}$, *we have, writing* $P = (p, 1)$

(a) $J(p, \{w^i\}).P = 0$;

(b) $P^T.J(p, \{w^i\}) = -Z^T(p, \{w^i\})$; *and hence,*

(c) $P^T.J(p, \{w^i\}) = 0$ *if* $P \in \Psi(\{w^i\}) \subset \mathcal{E}$

The first is the homogeneity of degree zero in the prices; the second follows from differentiating the expression for Walras Law; and the last one follows from the second using the definition of an equilibrium. It is clear therefore that the matrix $J(p, \{w^i\})$ is singular at any configuration $(p, \{w^i\})$ and the matrix $J(p, \{w^i\}) + J^T(p, \{w^i\})$ is singular if $P \in \Psi(\{w^i\}) \subset \mathcal{E}$; it may or may not be so elsewhere (i.e., out of equilibrium). Our final requirement may now be stated:

7. $J(p, \{w^i\}) + J^T(p, \{w^i\})$ has rank $n - 1$ whenever $P \in \Psi(\{w^i\}) \subset \mathcal{E}$.

Define $\mathcal{P} = [\{w^i\} \in \mathcal{W} :\sim \exists\{\bar{w}^i\} \in \mathcal{W}$ such that $U^i(\bar{w}^i) \geq U^i(w^i) \forall i$ with strict inequality for at least one i]: the set of Pareto Optimal distributions. We have the following:

4 $x^T.(J(p, \{w^i\}) + J^T(p, \{w^i\})).x \leq 0 \ \forall x \neq 0$, *with the inequality strict if* $x \neq \alpha P$ *whenever* $(p, 1) = P \in \Psi(\{w^i\}) \subset \mathcal{E}$ *and* $\{w^i\} \in \mathcal{P}$.

Proof: The proof will be in two stages. The first part involves showing that at a Pareto Optimal allocation $\{\bar{w}^i\}$, if $\bar{P} \in \Psi(\{\bar{w}^i\}) \subset \mathcal{E}$ then we have $\bar{P}^T.Z(P, \{\bar{w}^i\}) >$

$0\ \forall P \neq \alpha\bar{P}$; consequently the expression $f(P) = \bar{P}^T.Z(P, \{\bar{w}^i\})$ attains a minimum at $P = \bar{P}$ and hence, at $P = \bar{P}$, the hessian matrix of the function $f(P)$, $\nabla^2 f(\bar{P})$ must be positive semi-definite. Some tedious calculations establish that $-\nabla^2 f(\bar{P}) = (J(\bar{p}, \{\bar{w}^i\}) + J^T(\bar{p}, \{\bar{w}^i\}))$ where $\bar{P} = (\bar{p}, 1)$. The claim then follows by invoking Assumption 7. In fact, the first part follows directly from an Arrow and Hurwicz Theorem (1958) which shows that if the distribution of endowments $\{\bar{w}^i\}$ is Pareto Optimal, then the Weak Axiom of Revealed Preference holds i.e.,$\bar{P}^T.Z(P, \{\bar{w}^i\}) > 0\ \forall P \neq \alpha\bar{P}$, where $\bar{P} \in \Psi(\{\bar{w}^i\}) \subset \mathcal{E}$. So we have that the function $f(P)$ defined above attains a minimum at $P = \bar{P}$. Now we observe, using Claim 3,(b), that

$$f_k(P) = \sum_j \bar{P}_j Z_{jk}(P, \{\bar{w}^i\}) = -Z_k(P, \{\bar{w}^i\}) \Rightarrow f_k(\bar{P}) = 0;$$

further we note, again using Claim 3 (c), that

$$f_{kr}(p) = \sum_j \bar{P}_j Z_{jkr}(p, \{\bar{w}^i\}) \Rightarrow f_{kr}(\bar{p}) = -[Z_{rk}(\bar{p}, \{\bar{w}^i\}) + Z_{kr}(\bar{p}, \{\bar{w}^i\})]$$

so that $-\nabla^2 f(\bar{P}) = (J(\bar{p}, \{\bar{w}^i\}) + J^T(\bar{p}, \{\bar{w}^i\}))$ and hence positive semi-definiteness of $\nabla^2 f(\bar{P})$ implies that $(J(\bar{p}, \{\bar{w}^i\}) + J^T(\bar{p}, \{\bar{w}^i\}))$ is negative semi-definite; the matrix has rank $n-1$ by virtue of Assumption 7 and we know that $(J(\bar{p}, \{\bar{w}^i\}) + J^T(\bar{p}, \{\bar{w}^i\})).\bar{P} = 0$; so since $x^T.(J(\bar{p}, \{\bar{w}^i\}) + J^T(\bar{p}, \{\bar{w}^i\})).x \leq 0\ \forall x \neq 0$ and equality implies that $(J(\bar{p}, \{\bar{w}^i\}) + J^T(\bar{p}, \{\bar{w}^i\})).x = 0$, the rank condition implies that $x = \alpha\bar{P}$ and the claim follows. •

We show next that it is possible to drop the requirement that $\{\bar{w}^i\}$ is Pareto Optimal and still deduce the above claim. In other words,

5 $x^T.(J(\bar{p}, \{w^i\}) + J^T(\bar{p}, \{w^i\})).x \leq 0\ \forall x \neq 0$, *with the inequality strict if* $x \neq \alpha\bar{P}$ *whenever* $(\bar{p}, 1) = \bar{P} \in \Psi(\{w^i\}) \subset \mathcal{E}$ *for any* $\{\bar{w}^i\} \in \mathcal{W}, w^i > 0, \bar{P} > 0$.

Proof: Suppose to the contrary that for some $\{w^i\} \in \mathcal{W}$, $\bar{P} = (\bar{p}, 1) \in \Psi(\{w^i\}) \subset \mathcal{E}$ we have the matrix $(J(\bar{p}, \{w^i\}) + J^T(\bar{p}, \{w^i\}))$ is **not** negative semi-definite; i.e., it has at least one positive characteristic root.

Let $\bar{y}^i$ solve for each i the following maximum problem:

$$\max_y U^i(y) \text{ subject to } \bar{P}^T.y \leq \bar{P}^T.w^i$$

Then $\bar{y}^i$ is the **demand** by i at the equilibrium $\bar{P}$; and $\{\bar{y}^i\} \in \mathcal{P}$, a Pareto Optimal allocation. Note that for any $\{\bar{w}^i_\alpha\}$, where $\bar{w}^i_\alpha = \alpha w^i + (1-\alpha)\bar{y}^i$, $0 \leq \alpha \leq 1$, since $\bar{P}^T.\bar{w}^i_\alpha = \bar{P}^T.w^i \; \forall \alpha \in [0,1]$, demands at prices $\bar{P}$ remain unaltered and hence $\bar{P} \in \Psi(\{\bar{w}^i_\alpha\})$ for any value of $\alpha \in [0,1]$.

Note that $(J(\bar{p}, \{\bar{y}^i\}) + J^T(\bar{p}, \{\bar{y}^i\}))$ is negative semi-definite with rank $n-1$ i.e., there are $n-1$ negative characteristic roots and a single zero characteristic root.

Define

$$\bar{\alpha} = \sup_{\alpha \in [0,1]} \{\alpha : (J(\bar{p}, \{w^i_\alpha\}) + J^T(\bar{p}, \{w^i_\alpha\})) \text{ has } n-1 \text{ negative roots } \}$$

The supremum exists since by assumption for $\alpha = 1$, the relevant matrix has a positive root and hence has less than $n-1$ negative roots; for $\alpha = 0$ there are $n-1$ negative roots. Thus the set is non-empty since 0 belongs to the set and bounded above < 1. It is clear that for $\alpha = \bar{\alpha}$ the matrix $(J(\bar{p}, \{w^i_\alpha\}) + J^T(\bar{p}, \{w^i_\alpha\}))$ has $n-2$ negative roots with 0 as a repeated root; since otherwise, a slightly larger value for α would also be eligible. But this means that $(J(\bar{p}, \{w^i_\alpha\}) + J^T(\bar{p}, \{w^i_\alpha\}))$ has rank $n-2$ at $\alpha = \bar{\alpha}$[16]: this contradicts Assumption 7 since $\bar{P} = (\bar{p}, 1) \in \Psi(\{w^i_{\bar{\alpha}}\}$, as we discussed above. Hence there can be no such $\{w^i\} \in \mathcal{W}$, $\bar{P} = (\bar{p}, 1) \in \Psi(\{w^i\}) \subset \mathcal{E}$.

•

Thus note that the above means that

6 *For any $\{w^i\} \in \mathcal{W}$ and any $P \in \Psi(\{w^i\}) \subset \mathcal{E}$, we must have $(J(\bar{p}, \{w^i\}) + J^T(\bar{p}, \{w^i\}))$ negative semi-definite with rank $n-1$. Thus given any $\{w^i\} \in \mathcal{W}$, $\bar{P} = (\bar{p}, 1) \in \Psi(\{w^i\})$ is locally asymptotically stable under a process such as (3) and hence for every $\{w^i\} \in \mathcal{W}$ there is a unique equilibrium $\bar{P} = (\bar{p}, 1) = \Psi(\{w^i\})$.*

[16]This deduction may be made only because the relevant matrix is symmetric.

Proof: We observe that given some $\{w^i\} \in \mathcal{W}$, $\bar{P} = (\bar{p}, 1) \in \Psi(\{w^i\} \subset \mathcal{E}$, we have shown that $(J(\bar{p}, \{w^i\}) + J^T(\bar{p}, \{w^i\}))$ negative semi-definite with rank $n-1$; since $\{w^i\}$ will remain fixed we shall drop this from the arguments of the matrices and simplify notation further by writing $J(\bar{p}) + J^T(\bar(p)) = B(\bar{p})$, say. Now to verify local asymptotic stability of the equilibrium $\bar{P}$ under the process (3), we linearize this process around the equilibrium and we get

$$\dot{x} = \Lambda.\bar{J}(\bar{p})x \tag{4}$$

where $x \in \Re^{n-1}$, $x = p - \bar{p}$, Λ is a diagonal matrix of order $(n-1)$ with λ_j in the $jj-th$ entry. Further $\bar{J}(\bar{p})$ is the first $n-1$ rows and columns of $J(\bar{p})$. Notice that $\bar{J}(\bar{p}) + \bar{J}^T(\bar{p})$ must be negative semi-definite being a principal minor of $J(\bar{p}) + J^T(\bar{p})$ and in fact must be negative definite, since otherwise rank of $J(\bar{p}) + J^T(\bar{p}) \leq (n-2)$: a contradiction to Assumption 7. Now consider $V(t) = \sum_j x_j^2(t)/\lambda_j$ where $x(t)$ is the solution to (4). Note that $\dot{V}(t) = 2x(t)^T.\bar{J}(\bar{p}).x(t) = x(t)^T.[\bar{J}(\bar{p}) + \bar{J}^T(\bar{p})].x(t) < 0$ unless $x(t) = 0$; this allows us to conclude that $x(t) \to 0$ and hence that the equilibrium is locally asymptotically stable. Since this is so for every equilibrium $\bar{p}$ such that $(\bar{p}, 1) \in \Psi(\{w^i\})$, one may use a theorem of Arrow and Hahn (1971) to conclude that $\Psi(\{w^i\})$ is a function and that the equilibrium is unique given $\{w^i\}$. •

Finally note that we have on the basis of our assumptions shown that there is a unique equilibrium which is locally asymptotically stable under the process (3). There is another point which needs to be noted and this relates to the situation when the condition 7 is violated. Notice now that unstable positions of equilibrium are possible. In particular suppose that at some $\{w^i\} \in \mathcal{W}$, $\bar{P} \in \Psi(\{w^i\}) \in \mathcal{E}$, let $\bar{P}$ be unstable i.e., the matrix $J(\bar{p}) + J^T(\bar{p})$ has a characteristic root which is non-negative. (As for example in the Gale example, this was positive). If the demands at this equilibrium are given by the array $\{y^i\}$ and if the rank of $(J(\bar{p}, \{y^i\}) + J^T(\bar{p}, \{y^i\}))$ is full (i.e., $n-1$) then a redistribution of the endowments, as in the case of the Gale example, will lead to a stable equilibrium; and one need not eliminate all trade to arrive at a stable equilibrium.

Consider then the considerable weakening of assumption 7:

8. $J(p, \{w^i\}) + J^T(p, \{w^i\})$ has rank $n-1$ whenever $P \in \Psi(\{w^i\}) \subset \mathcal{E}$ and $\{w^i\} \in \mathcal{P}$.

We may state this conclusion thus:

7 *Under assumption 8, if for any $\{w^i\} \in \mathcal{W}$ the associated equilibrium P is unstable then there is a redistribution of the endowments $\{w^{i\prime}\} \in \mathcal{W}$ which would maintain the same P as equilibrium and for which P is locally asymptotically stable and there is some trade at the equilibrium prices.*

Thus in the above, the condition 7 has been weakened considerably: now we require that this be satisfied only at zero trade equilibria. The proof follows since if at the original distribution of endowments, $\{y^i\}$ denotes the array of demand at the equilibrium P, we know that $J(p, \{y^i\}) + J^T(p, \{y^i\})$ is negative semi-definite and hence there would be some redistribution lying on the convex combination of $\{y^i\}$ and $\{w^i\}$ which yields the desired outcome and such redistributions need not necessarily be the demand array.

To deduce global stability however, we need to strengthen the Assumption 7 to the following:

9. $J(p, \{w^i\}) + J^T(p, \{w^i\})$ has rank $n-1$ for all $P = (p, 1) > 0$.

Notice that we now require the rank condition to hold for all positive prices and not merely at equilibria. With this strengthening, we may show that

8 *Under the above Assumption 9, given $\{w^i\}$ the unique equilibrium $\bar{P} = (\bar{p}, 1)$ is globally asymptotically stable under the process (3).*

A6 Incentive for Competitive Transactions

A6.1 An Example

There are three individuals A, B, C with utility functions and endowments involving three goods x, y, z as under

$$A : \min(x, y) \ ; \ (1, 0, 0). \ B : y.z \ ; \ (0, 1, 0). \ C : \min(z, x) \ ; \ (0, 0, 1)$$

Thus although there are three goods each individual is interested in consuming only two goods and further, each has a stock of only one commodity. We first compute for the competitive equilibrium in this set-up. The first step involves the computation of demand functions. For A, we have $x = y$ and hence from the budget constraint $p.x + q.y = p$ where p, q are the prices of goods x, y relative to good z, which we consider to be the numeraire; thus demand function for A are given by:

$$x = y = \frac{p}{p+q}.$$

For B, the budget constraint is given by $q.y + z = q$, hence the first order conditions imply that $z = q.y$ and hence the demand functions for B are given by:

$$y = \frac{1}{2}; \ z = \frac{q}{2}.$$

For the individual C, we have the budget constraint $p.x + z = 1$ and we must have $z = x$ so that the demand functions for C are given by:

$$x = z = \frac{1}{p+1}.$$

Thus the excess demand function for good x denoted by Z_x is given by:

$$Z_x \equiv \text{ Aggregate Demand - Aggregate Supply} = \frac{p}{p+q} + \frac{1}{p+1} - 1 = \frac{p(1-q)}{(p+q)(p+1)};$$

similarly the excess demand for good y is given by:

$$Z_y = \frac{p}{p+q} + \frac{1}{2} - 1 = \frac{p-q}{2(p+q)}.$$

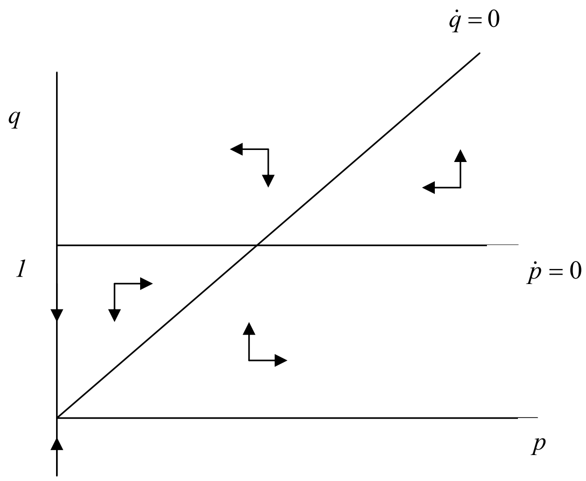

Figure 7: Competitive Equilibrium

Thus the **unique interior** equilibrium is given by $p* = 1, q* = 1$. We look at the trades which take place at equilibrium, next. The arrows in the diagram show the direction of price movements in dis-equilibrium when the dynamics is specified by a classical tatonnement discussed above; global stability is more tedious to establish but local stability of the interior equilibrium, for a process such as (3) i given from the characteristic roots of the Jacobian, evaluated at $(1, 1)$:

$$\begin{pmatrix} 0 & -1/16 \\ 1/16 & -1/16 \end{pmatrix};$$

the trace being negative and the determinant being positive signifies local asymptotic stability. But this aspect is of secondary concern at the moment. We are interested in what transactions take place at equilibrium.

A7 Trades at Competitive Equilibrium and Possible Deviations

A gets 1/2 units of good z from C in exchange for 1/2 units of good x; which is then traded to B for 1/2 units of good y.

Suppose now at the competitive equilibrium, B does not surrender 1/2 units of good y and say, surrenders only $1/2 - \epsilon$ units of y, $\epsilon > 0$; then B ends up with the bundle 1/2 units of x and $1/2 + \epsilon$ units of y with utility $1/4 + 1/2.\epsilon$ which is more than 1/4 the utility enjoyed at the equilibrium and hence B will have an incentive to undersupply.

Consider then the probability of being **undetected**; let this be given by $\theta = f(\epsilon)$, with $f'(\epsilon) \leq 0, f(0) = 0$, say; if detected undersupplying, a penalty of say $\tau = 2\epsilon$ units of good y is imposed on B. Thus B's expected utility is given by:

$$V(\epsilon) = \theta.[1/2(1/2 + \epsilon)] + (1 - \theta)[1/2(1/2 - \epsilon)] = 1/4 + 1/2\epsilon(2\theta - 1)$$

Notice that

$$V'(\epsilon) = 1/2(2\theta - 1) + \epsilon f'(\epsilon);$$

Hence it follows that $V'(0) = -1/2$ and B will stick to the straight and narrow and behave competitively.

However note that with the assumption on the function $f(\epsilon)$, we are in fact assuming that the probability of being **detected** is always 1; in case this is not so, as may be expected then notice that $f(0)$ is the highest probability of being undetected and suppose that this is $\bar{\theta} > 0$; then for small deviations, the probability of being detected is of the order of $1 - \bar{\theta} < 1$ what happens then ? Assume then that the penalty is some factor $\lambda > 1$ of the amount held back ϵ,($\lambda = 2$ in the above). Now

$$V(\epsilon) = 1/4 + (\lambda.\theta - (\lambda - 1))\epsilon/2;$$

and consequently, recalling that $\theta = f(\epsilon)$, we have

$$V'(\epsilon) = (\lambda f(\epsilon) - (\lambda - 1))/2 + \lambda f'(\epsilon).\epsilon/2.$$

Now

$$V'(0) = (\lambda.\bar{\theta} - (\lambda - 1))/2$$

which is negative only when

$$\bar{\theta} < \frac{\lambda - 1}{\lambda}.$$

Thus **competitive behavior is seemingly ensured, only when the probability of being detected is high enough in comparison with the penalty rate**.

Before passing on to other matters, it should be noted that neither A nor C have any incentive to renege on the competitive transactions. It is individual B who has an incentive to deviate and whose behavior needs to be regulated.

A8 Any Other Options for restoring Competitive Behavior?

What seems easy to state actually opens up an entire Pandora's box: regulation of competitive markets actually creates further problems as a moment's reflection will clarify. Since regulators may lack proper incentives to carry out the task assigned to them, insisting on regulators may have unleashed a whole set of new and additional problems and competitive behavior seems almost impossible to ensure in any meaningful way, eve in the context of such a simple example. Consider however the following option of redistributing endowments so that A has $(0, 1, 0)$ and B has $(1, 0, 0)$ while C continues to hold $(0, 0, 1)$: notice that as in the case of the Gale example considered earlier, at the equilibrium, purchasing power has not changed implies that neither has demands; however, transactions have altered substantially.

With the changed endowments, routine calculations lead to the following excess demand functions:

$$Z_x = \frac{q - p^2}{(p+1)(p+q)} \text{ and } Z_y = \frac{p(p-q)}{2q(p+q)}$$

Hence the situation is as depicted in the figure below:

The phase diagram shows that the new equilibrium, is the old equilibrium $(1, 1)$; the phase diagram may be exploited to show that this equilibrium is globally asymptotically stable under the usual price adjustment process; the demands at the new equilibrium, however remains as before:

$$A : (1/2, 1/2, 0);\ B : (0, 1/2, 1/2);\ C : (1/2, 0, 1/2);$$

however the transactions are different now: B sells $1/2x$ to A to get $1/2y$ and sells $1/2x$ to C to get $1/2z$; notice that now B too has no incentive to go back on the competitive

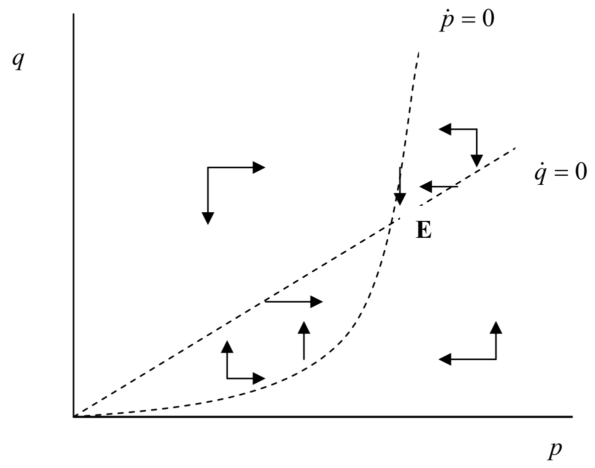

Figure 8: The Equilibrium

transactions. Thus a redistribution of endowments has succeeded where other measures such as regulation would have led to many other associated problems: creating more problems for the establishment of competitive transactions.

A9 Related Literature and Implications of the above exercises

Our results are a first step in the direction outlined at the beginning. The assumption 7 or the assumption 9 may appear too strong but they allow hopefully clean proofs. There is also another reason; weaker conditions may not yield the desired conclusion. Consider, for example the Scarf example. One may compute the matrix $\bar{J}(\bar{p}) + \bar{J}^T(\bar{p})$ matrix and show that it is the null matrix; the crucial Assumption 7 is violated[17]. For the Gale example, recall our analysis; the function $f(p, 2/3) = 0 \forall p$ implies that when the endowments $(2/3, 1/3)$ for A and $(1/3, 2/3)$ for B, the excess demand curve is horizontal nd hence every price is an equilibrium: the crucial assumption 7 is violated once again.

[17] See, for example, Mukherji (2007)

Consequently weakening this assumption may not be the route to follow. However we have also indicated a weakening: that is to require that the rank condition hold only at zero trade equilibria. We saw that this allowed us to conclude that there is a way of obtaining stability by redistributing endowments. Additionally, we need not redistribute to arrive at a zero trade equilibrium.

We should point out that there have been some related studies which try to investigate the results that may be obtained by aggregating across individuals. Two such works are due to Hildenbrand (1983) and Grandmont (1992). The result of the former, market demand having a quasi-negative definite Jacobian (identical to $J + J^T$ being negative definite with rank $n - 1$) is obtained by aggregation **only if** endowments are collinear. The second later study considers agents' characteristics in terms of a pair: preferences and income; the starting point of this analysis is a transformation indexed by $\alpha = (\alpha_1, \cdots, \alpha_n)$[18] of the commodity space. Consequently agents characteristics are expressed in terms of a marginal distribution over the space of preferences and income and for each preference and income, a conditional distribution over all transforms α. If every commodity is desired in the aggregate (a version of our boundary assumption) and if the conditional distribution over all transforms, given a preference and income, has a density which is flat enough then aggregate demand has very nice properties as for example gross substitution on a set of prices whose size is shown to depend on the degree of behavioral heterogeneity (the density being flatter implies increased heterogeneity).

We show that the assumption 7 implies that every equilibrium is locally asymptotically stable. A global version 9 implies global stability; we show that weakening these assumptions may not work but that if we have a very weak version of the rank condition then redistributing endowments will lead to stability. Our result, thus, is reminiscent of the Second Fundamental Theorem of Welfare Economics as we mentioned earlier; more accurately, a dynamic version of it: obtain stability through redistribution of endowments. The direction of research to support the theoretical foundations of competitive markets in economies with diversity, has to encompass these approaches described above.

[18] The axis corresponding to good j is stretched by e^{α_j}.

What about the transactions at competitive equilibrium? Notice there too, a redistribution of endowments may work much better than regulation, in the example at hand. The kind of redistribution that works is also revealing: when individual B sells some thing that is of no direct value to himself, there is no point in short sales. That seems to indicate that whenever the value system does not allow persons to cheat they do not. In the context of the example this was achieved by effecting a redistribution. Generally this may be difficult but then regulation opens up another can of worms. For what incentives do regulators have to carry out their jobs satisfactorily? This seems to indicate that a value system which enables agents to self-regulate will be helpful. This is a surprising conclusion given that much of theory of competitive markets is taken to be value-free; it is also generally assumed that the distribution of endowments just does not matter. Our analysis reveals that this again, is not quite accurate. For diverse preference patterns, particularly, these aspects assume significance.

References

C. M. Anderson, C. R. Plott, K-I. Shimomura, and S. Granat, (2004), Global Instability in Experimental General Equilibrium: The Scarf Example, **Journal of Economic Theory**, 115, 209-249.

K.J. Arrow and F.H. Hahn, (1971) **General Competitive Analysis,** Holden-Day: San Francisco.

K. J. Arrow and L. Hurwicz, (1958), On the Stability of Competitive Equilibrium I, **Econometrica**, 26, 522-552.

V. Bala, (1997), A Pitchfork Bifurcation in the Tatonnement Process, **Economic Theory,** 10, 521-530.

D. Gale (1963), A Note on the Global Instability of Competitive Equilibrium, **Naval Research Logistics Quarterly**, 10 81-87.

W. M. Gorman, (1961), On a Class of preference fields, **Metroeconomica** , 13, 53-56.

J-M. Grandmont, (1992), Transformations of the Commodity Space, Behavioral heterogeneity and the Aggregation Problem, **Journal of Economic Theory**, 57, 1-35.

J. R. Hicks, (1946), **Value and Capital** (2nd Edition), Clarendon Press, Oxford.

W. Hildenbrand, (1983), On the Law of Demand, **Econometrica**, 51, 997-1019.

M. Hirota, (1981), On the Stability of Competitive Equilibrium and the Patterns of Initial Holdings: An Example, **International Economic Review,** 22, No. 2, 461-470.

M. Hirota, (1985), Global Stability in a Class of Markets with Three Commodities and Three Consumers, **Journal of Economic Theory,** 36, 186-192.

M. Hirota, M. Hsu, C. Plott and B. Rogers (2005), Divergence, Closed Cycles and Convergence in Scarf Environments: Experiments in the Dynamics of General Equilibrium Systems, Caltech Social Science Working Paper, 1239, October 2005.

A. Mukherji, (1973), On the Sensitivity of Stability Results to the Choice of the Numeraire, **Review of Economic Studies,** , XL,427-433.

A. Mukherji (2000), Non-linear Dynamics with applications to economics: Stability of Competitive equilibrium reconsidered, **Journal of Quantitative Economics** 16 , 93-144.

A. Mukherji (2007), Global Stability Conditions on the Plane: A General Law of Demand, **Journal of Economic Theory** Vol. 134, May 2007, 583-92.

A. Mukherji (2008 a), The Stability of a Competitive Economy: A Reconsideration, **International Journal of Economic Theory**, June 2008, Vol. 4, 317-336.

A. Mukherji (2008 b), Competitive Markets and Diverse Economic Agents, presented at the Kobe COE Workshop,*Economics of Diversity: Issues and Prospects* at Kobe University, March 21, 2008.

T. Negishi (1962), Stability of a Competitive Economy, **Econometrica**, 30, 635-669.

H. Scarf (1960), Some Examples of Global Instability of the Competitive Equilibrium, **International Economic Review** 1 , 157-172.

L. Walras,(1877), **Elements d'economie politique pure**, Corbaz, Lausanne; translated as **Elements of Pure Economics** by W. Jaffe (1954), Allen and Unwin, London.

Abstract (in Japanese)

要約

本稿では、まず市場経済の安定性に関して定義し、選好が制限されないような安定性の条件が推定される。この推定では、エージェント間の多様性の存在が既知である経済を仮定している。その上で、均衡における超過需要関数についてのヤコビ行列の一階の条件が満たされる場合、均衡は局所的かつ漸近的に安定していることが示される。また、この条件が満たされない場合、どのような資源の再分配によって安定的な競争均衡が導かれるのか。さらには、重大なインセンティブ問題を引き起こす可能性のある罰則の代わりに資源の再分配を行うことで、さもなくば市場の失敗を発生させる可能性があるエージェントに的確なインセンティブを提供しうるだろうことを示す。

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